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ON THE THEORY OF QUANTUM MEASUREMENT

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Abstract

Many so called paradoxes of quantum mechanics are clarified when the measurement equipment is treated as a quantized system. Every measurement involves nonlinear processes. Selfconsistent formulations of nonlinear quantum optics are relatively simple. Hence optical measurements, such as the quantum nondemolition (QND) measurement of photon number, are particularly well suited for such a treatment. It shows that the so called "collapse of the wave function" is not needed for the interpretation of the measurement process. Coherence of the density matrix of the signal is progressively reduced with increasing accuracy of the photon number determination. If the QND measurement is incorporated into the double slit experiment, the contrast ratio of the fringes is found to decrease with increasing information on the photon number in one of the two paths.

1 Introduction

The Theory of Quantum Measurement has a long and venerable history. Many of the original discussions of the founders of quantum mechanics are contained in the reprint volume of Wheeler and Zurek[1]. Yet, inspite of its long history, the issues raised in these well known discussions have not been fully settled.

In this paper we attempt to make a modest contribution to this weighty problem. In doing so we are guided by a quote of Niels Bohr which reads: "... one sometimes speaks of "disturbance of phenomena by observation" or "creation of physical attributes to atomic objects by measurement." Such phrases, however, are apt to cause confusion, since words like phenomena and observation, just as attributes and measurements, are here used in a way incompatible with common language and practical definition. On the lines of objective description, [I advocate using] the word *phenomenon* to refer only to observations obtained under circumstances whose description includes an account of the whole experimental arrangement. In such terminology, the observational problem in quantum physics is deprived of any special intricacy and we are, moreover, directly reminded that every atomic phenomenon is closed in the sense that its observation is based on registrations obtained by means of suitable amplification devices with irreversible functioning such as, for example, permanent marks on a photographic plate, caused by the penetration of electrons into the emulsion" [Ref. 1, p. 3]. We have underlined the words that we consider particularly worthy of note. Bohr requires a description of the whole experimental arrangement. Further, if one is to state the outcome of the experiment in classical language, large amplification is required.

At the risk of making statements that may be considered even more controversial by the adherents of the Einsteinian school, we should like to strengthen Bohr's quote by saying: "*Physical reality* cannot be formulated until the measurement equipment used to determine the observables is specified and treated as a quantum system. The *large gain* of the measurement equipment provides the classical interface at the *output* of the measurement apparatus."

Much of the controversy involving quantum measurements is the consequence of the fact that it is very difficult to describe well the measuring equipment, according to our interpretation, to describe it quantum mechanically.

In quantum optics we have made great progress in describing optical components quantum mechanically. The theory has been well tested experimentally. The squeezing by a parametric amplifier is well understood theoretically and amply confirmed experimentally[2–6]. Less extensively explored, yet also tested, is the self-phase modulation and squeezing in optical fibers via the optical Kerr effect[7–9]. Hence it appears natural to use the well tested quantum description of optical devices to construct a measurement apparatus and test some of the predictions of quantum mechanics using such a measurement apparatus. This is the main objective of this paper. We start by describing a Quantum Nondemolition Measurement of the photon number of a signal via a nonlinear Mach-Zehnder interferometer. We follow the development of the composite wave function of the signal and measurement apparatus to the output. We shall see that the photon number in the signal can be determined with a negligible probability of error if the gain of the measurement apparatus is large enough. Further, when this is the case, the density matrix of the signal, obtained by tracing over the (Hilbert) coordinates of the measurement equipment, is diagonalized. Finally, since the probability of error of measuring a particular photon number approaches zero, each measurement, and not the whole ensemble, can be interpreted as yielding an interpretable result. This corresponds to the von Neumann projection operator interpretation. However, when the gain is not very large, the signal density matrix does not decohere, it is not diagonalized. This is consistent with Bohr's dictum that we can put the measurement results into classical language only if the gain of the measurement equipment is very large.

When no measurement is performed, and the signal and "measurement" beams are passed on into a second nonlinear Mach-Zehnder interferometer with a Kerr coefficient of opposite sign, the entire action of the first interferometer can be undone; the wave functions emerge disentangled! This confirms the reversibility of quantum mechanics.

We conclude with the double slit experiment. We put a nonlinear Mach-Zehnder measurement apparatus in each of the two light beams. As the accuracy of the photon number determination is systematically increased, the contrast of the interference fringes decreases accordingly.

2 The Quantum Nondemolition Measurement

Figure 1 shows a nonlinear Mach-Zehnder interferometer. The signal beam $\hat{a}_s^{in}$ at one frequency and the probe beam $\hat{b}^{in}$ at another frequency enter a Kerr medium through a dichroic mirror. At the end of the Kerr medium they are again separated by another dichroic mirror. A portion of the probe beam has been passed on directly for interference. Classically, the Kerr medium produces a phase shift on the probe beam that can be measured giving an indication of the intensity of the signal beam. Quantum mechanically, the process is described by the Hamiltonian of the Kerr medium[10]

$$\hat{H} = \hbar\kappa \hat{a}_s^\dagger \hat{a}_s \hat{b}^\dagger \hat{b} \tag{1}$$

where κ is a factor proportional to the Kerr coefficient; $\hat{a}_s$ is the annihilation operator of the signal photons, $\hat{b}$ that of the probe photons. They obey the usual commutation relations:

$$[\hat{a}_s, \hat{a}_s^\dagger] = 1 \tag{2}$$

$$[\hat{b}, \hat{b}^\dagger] = 1 \tag{3}$$

It should be noted that the Hamiltonian (1) does not account for a self-phase shift. This has been left out for convenience. A medium resonant at the sum frequency of signal and probe would be described by such a simplified Hamiltonian.

The two portions of the probe beam are combined by a beam splitter with the Hamiltonian:

$$\hat{H} = \hbar M[\hat{b}^\dagger \hat{c} + \hat{c}^\dagger \hat{b}] \tag{4}$$

As usual, one may consider the wave packets to evolve in time as they propagate along the system. If the beam splitter is 50/50, the parameter M must be chosen

$$\frac{M\ell}{v_g} = \frac{\pi}{4} \tag{5}$$

where ℓ is the length of the medium and v_g is the group velocity, ℓ/v_g is the travel time.

From the known Hamiltonian one may determine the evolution of the wave function $|\psi\rangle_s|\beta\rangle|0\rangle$ of the three input ports. They are products at the input, and become entangled at the output. We denote the output annihilation operators by $\hat{f}$ and $\hat{g}$. The balanced photodetector measures the expectation values of the difference current operator $\hat{I} = \hat{f}^\dagger\hat{f} - \hat{g}^\dagger\hat{g}$ and its moments[11].

$$\langle \hat{I} \rangle = |\beta|^2 \sin(\kappa \hat{a}_s^\dagger \hat{a}) \simeq \kappa |\beta|^2 \hat{a}_s^\dagger \hat{a}_s \tag{6}$$

The expectation value traced over the Hilbert space of the probe yields the sine of the signal photon operator. If the sine function can be expanded to first order, it becomes the photon operator. The mean square fluctuations follow from the second moment and are[11]

$$\langle |\Delta \hat{I}|^2 \rangle = |\beta|^2 \tag{7}$$

if the signal is in a photon number state. This is shot noise since $|\beta|^2$ is the photon number in the probe beam.

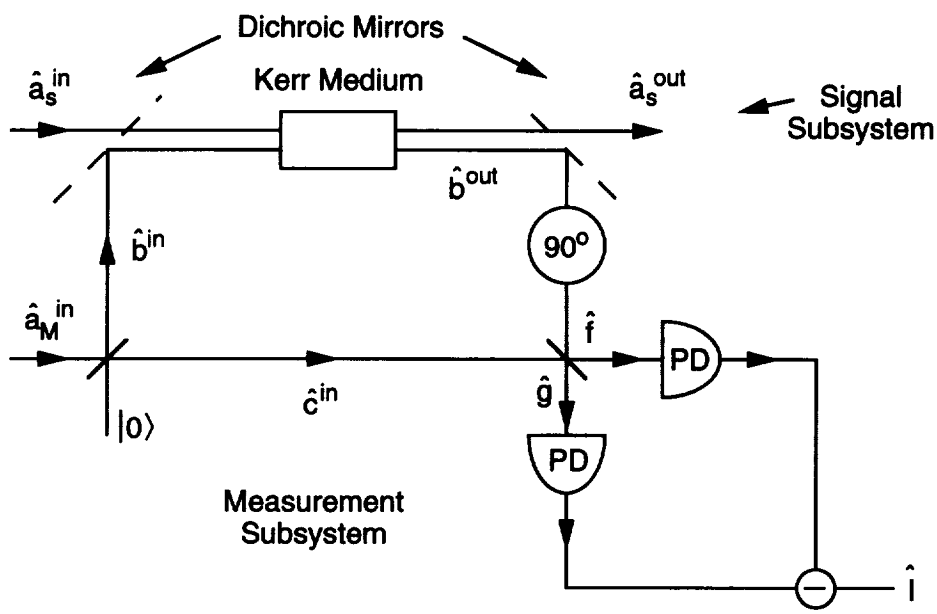

FIG. 1. Schematic of nonlinear Mach-Zehnder interferometer and balanced detector.

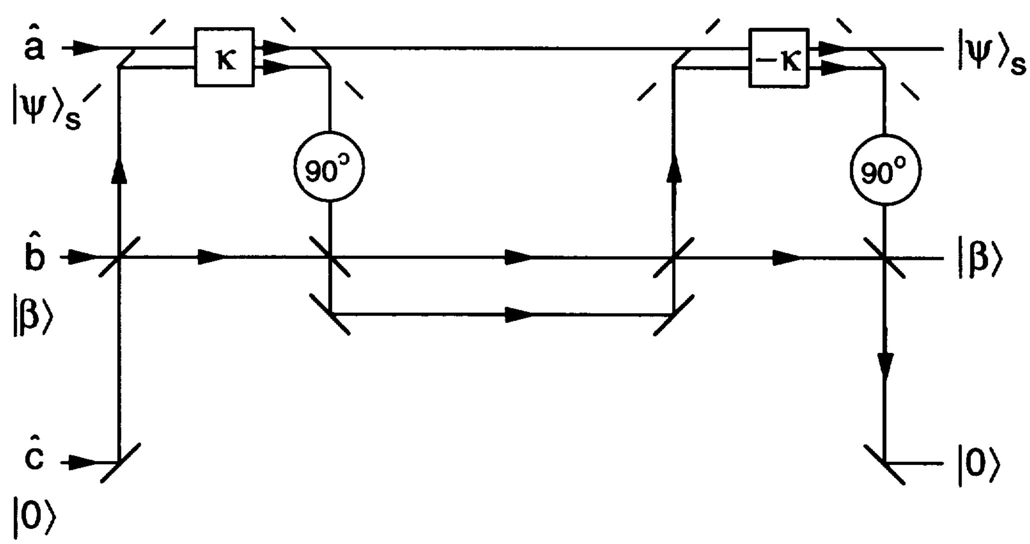

FIG. 2. Two nonlinear Mach-Zehnder interferometers with media of equal and opposite Kerr coefficients.

The probability of error follows from the mean square fluctuations (7) that approach gaussians in the large photon number limit[11]:

$$P_{\text{error}} \simeq 2\sqrt{\frac{2}{n}}\frac{1}{|\kappa\beta|}e^{-|\kappa\beta|^2/8} \tag{8}$$

If $|\kappa\beta|^2 \gg 1$, the probability of error can be made arbitrarily small. The physical meaning of this quantitity can be fathomed as follows. $\kappa|\beta|^2$ is the phase shift due to the probe photons, κ itself is the phase shift due to one photon. The geometric mean of these two products has to be made very large. If we used fiber interferometers, these operating parameters are not easily achieved. Here, however, we are not concerned with the practical realization of the measurement apparatus, but only with the theoretical conclusions that can be drawn from it. In particular, we find that the probability of error can be made arbitrarily small, for $|\kappa\beta| = 10$, it is 10^{-6}. This means that each measurement has vanishing error probability. Hence one may interpret every measurement, and not only the ensemble, as yielding a definite result. This is analogous to the von Neumann projection postulate which interprets a measurement as projecting the state into an eigenstate. Pursuing this interpretation further, we can say that a measurement with the Mach-Zehnder interferometer at large gain projects the signal into a photon state.

3 The Density Matrix

The trace of the density matrix over the measurement system part at the output of the signal-measurement system of Fig. 1 can be evaluated for a signal wave function[11]:

$$|\psi\rangle_s = \sum_n c_n|n\rangle \tag{9}$$

It is

$$\begin{aligned} \langle\rho_s\rangle &= \sum_{n=0}^{\infty} e^{-\frac{|\beta|^2}{2}}\frac{|\beta/\sqrt{2}|^{2n}}{n!}e^{i\kappa n\hat{a}^\dagger\hat{a}}|\psi\rangle_s^{\text{in}}\;{}_s^{\text{in}}\langle\psi|e^{-i\kappa n\hat{a}^\dagger\hat{a}} \\ &\simeq \sum_{m,\ell=0}^{\infty} \hat{c}_m^* c_\ell|m\rangle\langle\ell|\;\underbrace{e^{-\frac{|\kappa\beta|^2}{8}(m-\ell)}}_{\text{error probability}}\;e^{i\frac{|\beta|^2}{2}\kappa(m-\ell)} \end{aligned} \tag{10}$$

In the limit of large gain, the density matrix traced over the measurement equipment becomes diagonal at the same rate as the probability of error approaches zero (note the exponential factor!). Hence, again, we see that the signal acquires a classical (decohered) appearance when the gain of the measurement system ($|\kappa\beta|$) is made very large.

4 Reversibility

If one does not perform a measurement on the probe beam, but reintroduces it in the second Mach-Zehnder as shown in Fig. 2, which has a Kerr coefficient of opposite sign, one can disentangle entirely the wave functions. This shows, of course, the reversibility of quantum mechanics if no measurement intervenes in the process. Of course, no measurement could have been undertaken, because the probe beam was completely recycled. This brings us back to the act of measurement. A measurement is an irreversible process that prevents recycling. Indeed, in the present example the probe beam is passed into a balanced detector in which it is absorbed. Only then can one apply the homodyne photon detection formula to evaluate the current operator statistics.

5 Tracing, Decoherence and the Act of Measurement

The density matrix of the signal system becomes diagonal in the signal Hilbert space when traced over the probe space. Tracing is a mathematical operation which, according to the postulates of quantum mechanics, evaluates expectation values. In the context of the derivation of the signal density matrix, the reduced density matrix can be interpreted as a "Gedankenexperiment" on the density matrix of the signal after passage through the Mach-Zehnder. Accompanied by the statement that the signal and probe systems would never be combined again, the entanglement that in fact exists between the two systems could never be reversed. In this sense, the reversibility of quantum mechanics is broken. In an actual measurement, of course, the apparatus works on the probe subspace, causes partial or total decoherence in that space, and leads "de facto" to an irreversible action.

6 Two Slit Experiment

Finally, let us look at the "two-slit" interference experiment of Fig. 3. The two slits are here replaced by the two arms of an interferometer. A phase shifter in one of the arms changes the phase of the superimposed beams. If the two beams were perfectly coherent, the intensity at the detector would have to show perfect extinction. However, we mount two QND apparati in each of the arms to ascertain the number of photons passing through them individually. The gain of the apparati can be adjusted, thus changing the accuracy of the measurement of the photon number passing through each arm. One can then compute the expectation value of the contrast and finds it to be[11] (see Fig. 4)

$$\langle \hat{I} \rangle = e^{-|\kappa\beta|^2/4} \cos\theta \tag{11}$$

Thus, a similar exponential factor as the one that appears in the error probability determines the extinction of the contrast. The factor is squared, because two measurements are being performed. Here again we find that the transition between the behavior of the photon as a wave and that of a particle is a continuous one. The accuracy of the determination of the photon number determines how much the photon behaves as a particle.

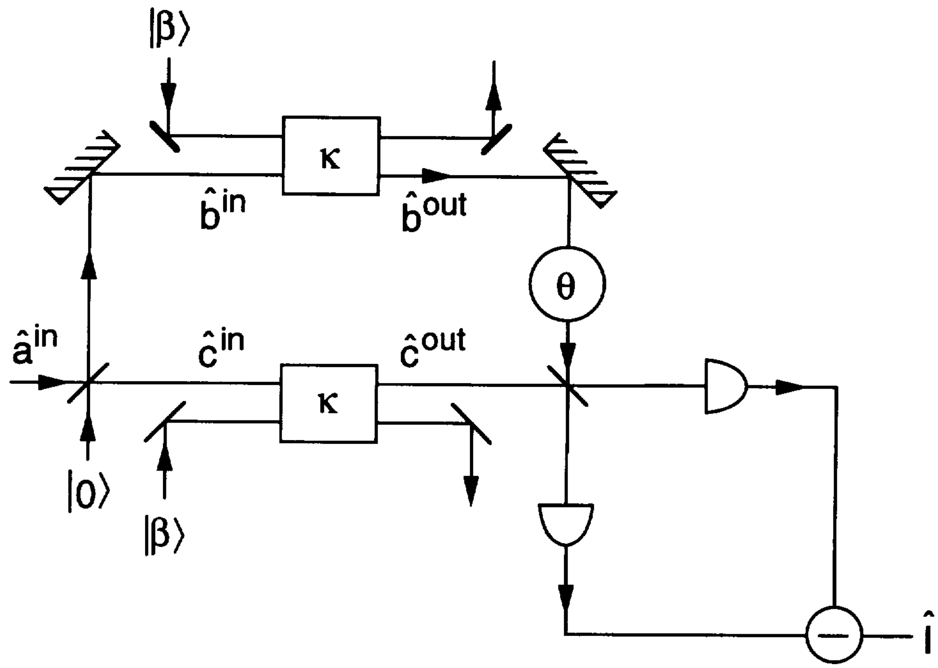

FIG. 3. An interferometer representing two-slit interference and attached QND measurement apparati.

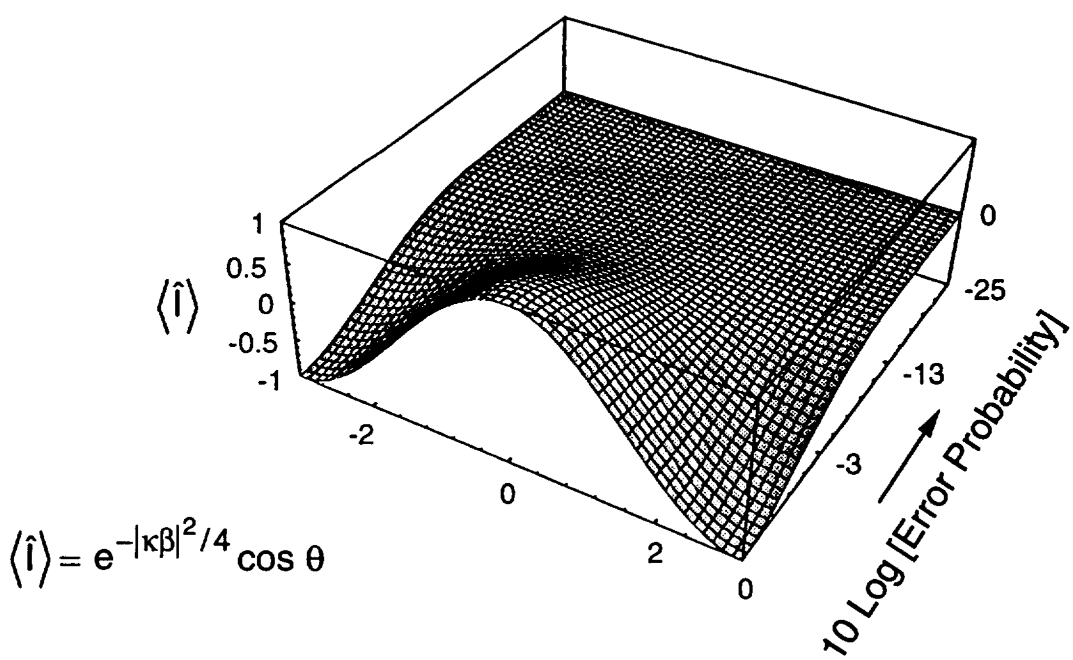

FIG. 4. Expectation value of detector current versus phase and error probability of photon number determination.

7 Conclusion

We started with the postulate that a proper formulation of a quantum measurement has to quantize the measuring apparatus as well. The quantum formalisms developed for optical components enable one to do a full quantum analysis of an optical measurement apparatus. The measurement apparatus of photon number with infinite gain yields results that can be described in classical language: photons behave as particles (since we chose a particle measurement apparatus). When the gain is not infinite, the behavior is more duplicitous, it is not what one would call the behavior of a classical particle. This confirms Bohr's statement that it is necessary to have large gain to obtain measurement results that can be put into classical language. We also found that a measurement with infinite gain is equivalent to a projection operation on the signal.

If no measurement is undertaken, the entanglement of the signal and probe states can be fully undone by an inverse apparatus.

Finally, the "double-slit" experiment can also be described in terms of partial knowledge of the photon number in each of the paths. If the knowledge is only partial, there can still be interference of the two beams.

8 Acknowledgments

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9 References

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